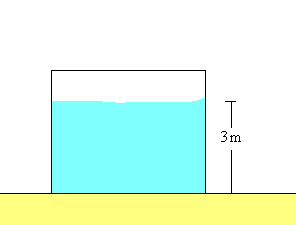
**Example**

What is the pressure at the bottom of a 3m deep swimming pool?



Just use the equation above. Note that the pressure at the top of the swimming pool is atmospheric pressure: Patm. = 101 kPa. And note that the density of water is 1000kg/m3. Then we have,



**Example**

Atmospheric pressure at the surface of the Earth is 101 kPA. If the atmosphere had a uniform density, what would this imply the height of atmosphere is? Use the fact that air has a density of 1.2kg/m3 at the surface of the Earth.

We can use the equation above again. At the top of the atmsophere, P0 would equal 0, since it be in space, which is a vacuum. Therefore, we’d have,



So the height of the atmosphere would be about h = 8.6 km, which is about 28 400 ft., which is around the height of Mount Everest. Clearly, the atmosphere does not, therefore, have a constant density.

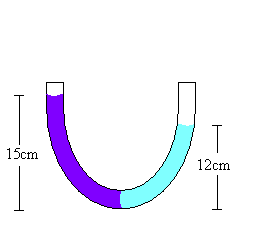
1. An airtight box has a removable lid of area 0.02m2 and negligible weight. The box is taken up a mountain where the air pressure outside the box is 0.87 x 105 Pa. The inside of the box is completely evacuated. What is the magnitude of the force required to pull the lid off the box?

The net force is:



**Example: Pressure gauges**

One way to determine the density of unknown liquids is with a U-tube. Consider the U-tube illustrated below, containing water and liquid X. What is the density of liquid X?



Well first of all, the density of water is 1g/cm3 (which translates to 1000 kg/m3). Secondy, the pressure at the bottom of the tube – at the X-water interface, we’ll call P. And we’ll note that the pressure at the top of the X column is Patm., and the pressure at the top of the water column is Patm. as well. Now going from the top of the X column to the interface, we must have that,



and going from the top of the water column to the interface we must have that,



equating these two we must have,

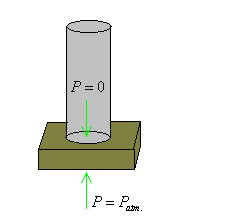


Note we wouldn’t set the pressures at the horizontal line crossing top of light blue liquid and middle of dark blue liquid equal because if were case, then gravity, acting on the two liquids would exert a net imbalanced force – more on the dark one since its denser, and this would push everything to the right.

The main idea is that the pressure at a point must be the same no matter how we get to that point – no matter what liquids we go through – not so much that the pressure at an ‘interface’ must be equal.

**Example**

Suppose you have a really good vacuum cleaner capable of creating a perfect vacuum. And suppose its nozzle has a 2cm diameter. What is the heaviest object you could pick up with the nozzle?



Well consider the nozzle on the ‘brick’. Inside the nozzle, there is a vacuum and so no atmospheric pressure. Outside the nozzle there is atmospheric pressure. The atmospheric pressure on the bottom, top (and sides) of the brick exert forces which cancel each other out, except below the cross section of the nozzle. Therefore the net force we have acting on the brick is:



This is the mechanism and octopus uses to open clam shells by the way. Its suckers create a partial vacuum at the surface of the clam shell. Inside the clam shell the pressure is that of the ambient water pressure. This gives rise to a net force pushing the clam shell apart from the inside.

2. The density of ice is 917 kg/m3, and the density of sea water is 1025 kg/m3. A swimming polar bear climbs onto a piece of floating ice that has a volume of 6m3. What is the weight of the heaviest bear that the ice can support without sinking completely beneath the water?



Using N2L on the ice we have:



**Problem 11**

What net force would you have to exert in order to hold a 2kg basketball (radius 10cm) filled with air under water? Do you have to push down or up?



1. Consider a recreational submersible at a depth of 100m. What is the net force on a window with dimensions 50cm×75cm if the pressure inside the submarine is maintained at atmospheric?

The pressure at a depth of 100m is:



Therefore the net force acting on the window is:



18. Suppose you’re in a submersible at a depth of 1000m below the ocean surface. If the pressure inside the submersible is atmospheric (Patm. = 1.01×105Pa), what force does the water exert on the submersible’s window which has dimensions 25cm×50cm? D

The force is given by:



19. A box with cross sectional area A = 4m2 and height 50cm has a density ρ = 750kg/m3. What is the largest mass that it can support without sinking? A

Look at the forces acting on the box.

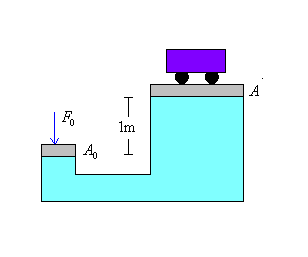


2. A block of wood has a mass of 5 kg and a density of 666 kg/m3. It is to be loaded with lead so that it will float in water with 80% of its volume submerged. What mass of lead is needed if the lead is attached to the top of the wood?

The net force on the block of wood must be equal to zero for it to float so we must have:



3. What force, F0, is required to hold the car (m = 1200kg) on the larger piston. Be sure to take into account the height difference of the two pistons. Let the cross section area A = 6.5m2 and A0 = 0.8m2.



The pressures P0 and P at the left and right cylinders respectively are given by the equation:



and so the forces are related by:



Now F must be mg = (1200)(9.8). Therefore F0 is:



4. You carry a garden hose with a nozzle of radius 2cm. And suppose that with this hose you can fill a cylindrical bucket with radius 12cm and height 15cm in 3s. How fast would the water come out of the hose if you covered 1/3 of the nozzle with your thumb?

From the first line we get the flow rate:



and then using the equation f.r. = Av we can get the velocity of the water,



5. A water pipe having a 2cm inside diameter carries water into the basement of a house at a speed of 1.2m/s and a pressure of 241 kPa. If the pipe tapers to 1.6cm and rises to the second floor 3m above the input point, what are the **(a)** speed and **(b)** water pressure (in kPa) at the second floor?

Use Bernoulli’s equation:



Filling in what we know so far…



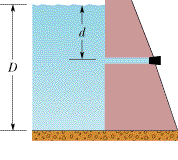
we don’t know P2 or v2. We can get v2 from the flow rate equation:



and now we can find P2. We have:



6. In the figure below, the fresh water behind a reservoir dam has depth *D* = 20 m. A horizontal pipe 5cm in diameter passes through the dam at depth *d* = 7m. A plug secures the pipe opening. If the plug is removed, what water volume exits the pipe in 4h?



First we must determine the speed of the water pouring out of the hole. This comes from Bernoulli’s equation:



and so the flow rate is:



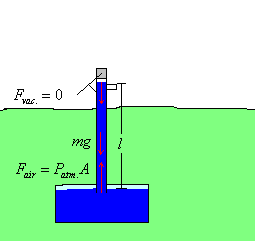
and so finally, the amount of water that flows out in 4 hours is:



**Example**

What is the maximum length of pipe, ℓ, that can be used in this manner to draw water from a reservoir?

Well (no pun intended), if you can evacuate all the air from the pipe, as aforementioned, then the pressure at the top of the pipe will be P = 0. The pressure at the bottom of the pipe will be Patm., due to the air there.



If the water column is to rise, then we need that the net force on it is greater than 0. So we need,



So we would need ℓ to be less than 10m or so. Any larger length of pipe, and the air pressure in the reservoir would be unable to hold the weight of the water in the pipe. And it is the case that for deeper wells, impellers, like that used in the vacuum cleaner have to be added to the pipe to help force the water up through it.

**Example:**

Suppose you have a hydraulic lift with A0 = 0.02m2, and A = 1m2. What force do you need to apply to A0 in order to lift a car with mass 1000kg?

We’d need to apply a force of:



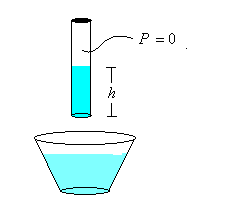
which correlates to about 45 lbs. This would be fairly easily done. Next question, if we want to raise the car a distance of 2m, how far would we have to push on our piston?

We can apply the concept that the work we apply to piston A0 should equal the work outputted to piston A. Therefore we must have,



So we see that even though we have to exert much less force, we have to apply that force over a much greater distance in effect. Thus the net work we do either way remains the same.

3. When you put a straw in cup of water (or Diet Coke), then put your finger over the top and lift it out of the cup, most of the liquid in the straw remains there. Determine what is the maximum height of water, h, that you could keep in a straw in this fashion, as shown below. For the sake of calculation, assume that there is a vacuum at the top of the straw and so the pressure is 0 there.



The upward force on the column of water (due to the pressure at the bottom of the straw) is:

.

The downward force on the column of water (due to the pressure at the top of the straw is:

 (since the pressure at the top is 0)

The weight of the column of water is:

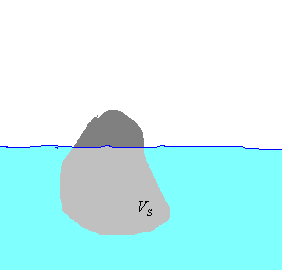


Setting the sum of the forces equal to zero, we have,



**Example**

How much of an iceberg is under water? i.e., what fraction, by volume, of an iceberg is below water?



Let Vs be the volume of the iceberg that is below water. And let V be its total volume. Since the iceberg is floating, it must be that the sum of the forces on it equals 0. Therefore we have,



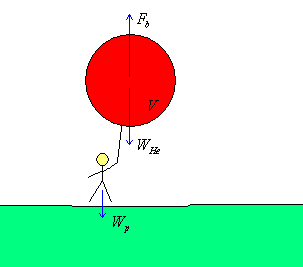
now the density of ice is about 920 kg/m3 while the density of saltwater is about 1030 kg/m3, and so we have,



So 89 % of an iceberg is below water.

**Example: How many helium-filled balloons are necessary to lift you off the ground?**

Let’s calculate the volume of a Helium balloon necessary to lift you off the ground. Air has a density of 1.2kg/m3. And He2 at room temperature has apparently a density of 0.18g/m3 which is negligible. And we’ll take your mass to be 70kg. The forces on you/balloon are shown below,



To achieve lift-off we need that the sum of forces acting on you/balloon are greater than zero. So



And so you would need around a balloon of volume 69m3 approximately.

5. Suppose that the density of the atmosphere decreases with height according to the equation given below. And imagine that a He (ρHelium = 0.18kg/m3) filled balloon of volume V = 125m3 is carrying a mass M = 100kg. To what maximum height can the balloon ascend?



The height to which it can ascend is that height to which the buoyant force matches the weight. This will be that height where,



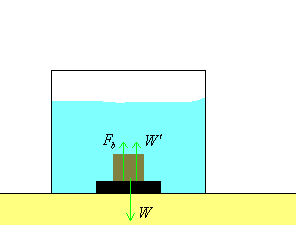
The height that corresponds to this density is:



**Example**

There is a nice way to measure density. If we know its density then we can often determine the composition of the object (this is how Archimedes determined that the crown the King of Syracuse gave him was not entirely gold I think). First determine the weight of an object, W, in air. And then determine the weight of the object W′ submerged in a fluid, such as water. How can we use these to determine the density of the object? and its volume? Determine these for the particular case where W = 600N, and W′ = 200N.

Well consider the object submerged in water, on a scale reading its weight (in water), W′. And label all the forces acting on it.



Add them up and set to 0 since it is not moving,



So solving for the object’s volume we have,



and now we can get its density by using,



which implies,



applying to our particular case, we’d have,

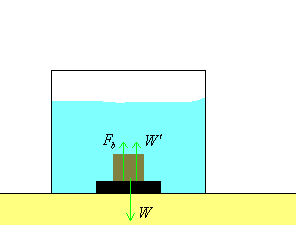


and,



4. Suppose that you stand on a scale in your living room and it reads 165 lbs. (735 N), and that if you completely immerse yourself in a pool of water and stand on the scale it reads 50 lbs. (222 N). What is your volume? Note that the reading on the scale is the force that it exerts on *you*. You should draw a free body diagram of all the forces acting on you at the bottom of the pool.

Well consider the object submerged in water, on a scale reading its weight (in water), W′. And label all the forces acting on it.



Add them up and set to 0 since it is not moving,



So solving for the object’s volume we have,

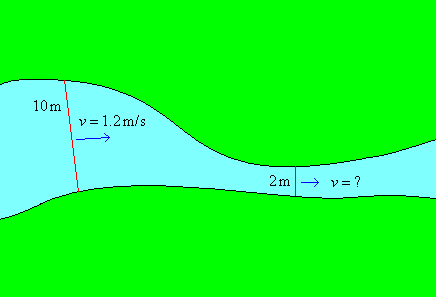


applying to our particular case, we’d have,



**Example**

Suppose a river narrows from 10m wide to 2m wide (maintaining the same depth). If the speed of the water at the 10m cross section is 1.2m/s, what is its velocity at the 2m cross section?



From the continuity equation, we have,



**Example**

Suppose you’re going to wash your car. You start by filling a bucket of water with volume V = 2×10-3 m3, in 5s. If the nozzle has a diameter of 1cm, how fast is the water coming out of the nozzle?

Well the rate at which water is coming out of the nozzle is:



and this flow rate is equal to f.r. = Av, where A is the cross section area of the nozzle, which is:



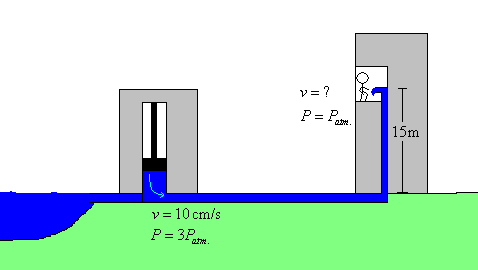
therefore we have,



which is about 12mph.

**Example**

Suppose the utility company maintains a pressure P = 3Patm. in the water line via the pump aforementioned. And suppose it pumps water with a velocity v = 10cm/s. How fast will the water come out of your faucet, if you’re located on 15m above the ground?



We know the pressure and velocity of the water at the pump in the line. And we know the pressure of the water at the tip of the faucet (this is atmospheric pressure b/c the water is open to the air). What we don’t know is the velocity, but we can use Bernoulli’s principle to obtain this,



**Example**

What is the power exerted by the pump in the previous example, if the diameter of the pipe is d = 20cm?

The power exerted by the pump is defined to be:



We’ll recall that this worked out to be:



Now for our case φ = 0˚ since the pump force and velocity of the liquid are aligned. Now the force the pump exerts is F = PA, where A is the cross section are of the pipe. And v = f.r./A, using the flow rate equation. So…

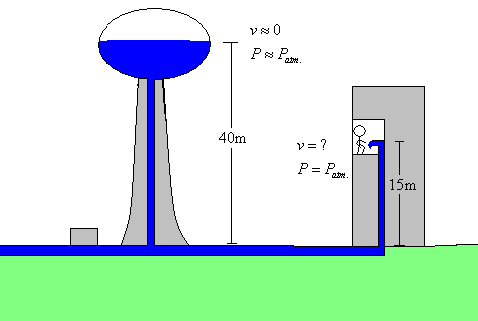


The flow rate is given by f.r. = Av, where A is the cross sectional area of the pipe, and v is the velocity of the fluid through the pipe. So filling in our values,



**Example**

Suppose a water tower is 40m high, and is supplying water to the building in the previous example. Further, suppose the water level in the water tower is dropping at a negligible speed (as it would since there is so much water), and also that the pressure inside the tank is atmospheric, as it is usually maintained to be. What then is the speed of the water coming out of the faucet?



According to Bernoulli’s equation we have,



**Example**

Suppose the water tower can hold a volume of water V =1000m3. How long would it take the 952 W pump in the previous example to fill the water tower all by itself.

We can use the power equation. The pump has to move a quantity of liquid from a height y = 0 to a height y = 40m. This will require an amount of work equal to the total change in gravitational potential energy of the water,

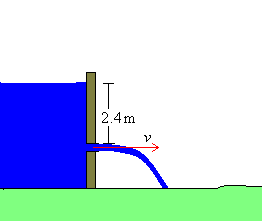


The pump operates with a power of P = 952 W, and so the time it will take to finish the job is:



**Example**

To generate hydroelectricity, one may construct a damn, and then construct a pipe that allows water to spill out of the damn, the kinetic energy of the falling water is then converted to electric potential energy (electricity) in much the same way as the angular kinetic energy of a windmill is converted to electricity (you’ll perhaps learn about these methods in physics 2). Suppose that that our pipe is located 2.4m below the damn. How fast does the water pour out of the damn. If the diameter of the pipe is 50cm, what power is delivered by the pipe, i.e., how much KE is flowing out of the pipe per second?



We will use Bernoulli’s equation. Let us consider point 1 to be at the top of the damn, and point 2 to be at the opening on the side. The pressure at points 1 and 2 is Patm. because at each point the water is exposed to the atmosphere alone. Further, the water at the top of the damn is moving very slowly, so its velocity is approximately 0. Proceding…



Observe that this is the same velocity as would be had if the water had fallen directly through a vertical distance of d.

The kinetic energy flowing out of the pipe, per unit time is:

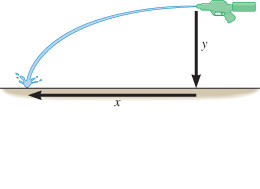


Now the flow rate is f.r. = Av = π(0.25)2(6.9) = 1.36 m3/s. Therefore the power is:



If the machinery transfering the kinetic energy to electric potential energy were perfectly efficient, then this damn would generate 32.4 kW of electrical power. This is a pretty small damn however. Hoover damn for instance is 750ft high, and generates around 2GW power.

2. A hand-pumped water gun is held level at a height of 1.3m above the ground and fired. The water stream from the gun hits the ground a horizontal distance of 7.5m from the muzzle. Find the gauge pressure of the water gun's reservoir at the instant when the gun is fired. Assume that the speed of the water in the reservoir is zero, and that the water flow is steady. Ignore both air resistance and the height difference between the reservoir and the muzzle.



First we can determine the velocity of the water out of the nozzle. From kinematics, the time to impact is:



and the x-velocity is given by:



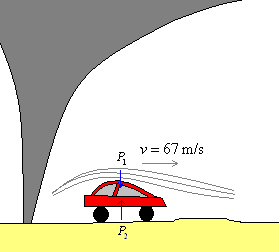
Now we must apply Bernoulli's equation to the liquid. Let point 2 be the nozzle and point 1 be the reservoir. Then we have:



Note we are using gauge pressures since we set P2 = 0.

**Example**

Suppose a tornado runs through your neighborhood, with winds of 150mph. Assume that the air at the bottom of your car has negligible speed. Assume the car is 1.5m tall, and suppose your car has a horizontal cross section aiea of A = 2.5m2. What is the heaviest car such winds could lift?



Well, let’s use Bernoulli’s equation. Let point 1 be on the top of your car, and point 2 be at the bottom of your car. The net upward force on the car would be:



We can calculate the pressure difference using Bernoulli’s equation.



Now P1 is not atmospheric pressure because the air is moving and as such, P1 is lower than atmospheric pressure, since moving fluids have lower pressures than their static counter parts. P2 is probably somewhere close to Patm. since the air beneath isn’t moving by assumption, but since we are only interested in the difference of pressures, this is not of particular concern. So, using the fact that the density of air is 1.2kg/m3, we have



and therefore the force on your hypothetical car would be:

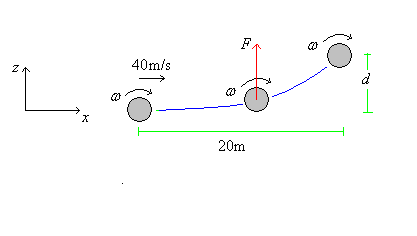


As you can see, the force can be rather large.

**Example: Curve Ball**

Suppose you’re a pitcher in a baseball game and you throw the ball with a speed of 40m/s, and spinning at a rate of 20 rev./s clockwise. Suppose it has a diameter of 6cm, and has a mass of 0.9 kg. How far will it curve after traveling to home plate 20m away?

To answer this question we’ll take a top-down view.



The ball is moving at a speed of 40m/s in the x-direction. And it is rotating at a speed of 20 rev/s = 20·2π rad/s = 126 rad/s. That means that the tangential speed of the ball’s surface is v = ωr = (126)(0.03) = 3.8m/s. Now observe carefully that the total speed of the surface of the ball near the top of the page will be 40 + 3.8 = 43.8m/s because the ball is traveling to the right, and the top surface is rotating that way too. And the speed of the surface of the ball near the bottom of the page will be 40 – 3.8 = 36.2m/s because the ball is traveling to the right, but the bottom surface is rotating towards the left. Now from the perspective of the ball, the air near the top of the page will be moving leftward at 43.8m/s, and the air at the bottom of the page will be moving leftward at 36.2m/s. Since the air at the top of the page will be moving faster than the air at the bottom of the page, there will be greater pressure at the bottom of the ball/page than at the top of the ball/page, and so there will be a force, F pointing towards the top of the page. We can determine this force using Bernoulli’s equation. Let point 2 be at the top of the ball/page, and point 1 be at the bottom of the ball/page. Then,



The net force on the ball is:



where A is the cross-sectional area of the ball. A = πr2 = π(0.03)2 = 2.8×10-3m2. And so F is:



The acceleration of the ball towards the top of the page would be, according to N2L,

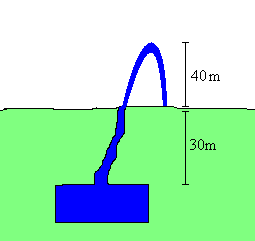


Now we need to know how long the force acts on the ball in order to determine how far the force moves the ball. The length of time the force acts is the amount of time it takes for the ball to travel the 20m. Now since the ball travels at 40m/s, it would take ½ second to complete the 20m distance. Now we can determine how far it deflects in the 0.5s using the simple kinematics equation,



Note that v0z is 0 because this v0z refers to the velocity in the z-direction, not in the x-direction. Observe how this answer makes physical sense, a 15cm deflection is within the range of possibility given our familiarity with baseball.

20. Consider a geyser like Old Faithful in Yellowstone national park. If the geyser is shoots water to a maximum height of 40m, and the depth of the ‘well’ from which the water originates is 30m, what is the water pressure in the well? You can approximate the velocity of the water in the well as 0. E



Using Bernoulli’s equation and letting point (1) be at the well, and point (2) be at the top of the geyser, we have:



The pressure acting on the well comes from the weight of the dirt above. Since the dirt isn’t moving, we can say:



Now using Bernoulli’s equation, comparing the point at the top of the well (1), and the point at the top of the parabolic arch (2), we can say



21. An airplane is flying through the air. Suppose its wings have a total cross-section of 10m2. If the air going over the top of the wings is traveling at a rate of 200m/s, and the air going over the bottom of the wings is traveling at a rate of 180m/s, what net upward force does the air exert on the wings? C

The force is given by the pressure difference on the wings,



we can figure out the pressure difference from Bernoulli’s equation:



So the force is:



4. A ship is floating on a lake. Its hold is the interior space beneath its deck; the hold is empty and is open to the atmosphere. The hull has a hole in it, which is below the water line, so water leaks into the hold. The hole is located 5.2 m beneath the surface of the lake. What is the speed with which water leaks into the hold?

Use Bernoulli’s equation. Let point 1 be the point at the surface of the water, and point 2 be the point at the leak (just inside the ship). Then,



8. The construction of a flat rectangular roof (6m×6m) allows it to withstand a maximum net outward force of 31 x 104 N. The density of the air is 1.29 kg/m3. At what wind speed will this roof blow outward?

The maximum allowable net pressure on the roof would be:



Again, use Bernoulli’s equation to relate the air at the top of the house to the air inside:

